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III.

Euler's Equations of Motion.

BY JAMES LOUDON, *University College, Toronto, Canada.*

1. A rigid body fixed at O has at time t rotations, ω_1, ω_2, ω_3 round the principal axes OA, OB, OC, to determine the changes per unit time in these rotations.

The positions OA', OB', OC' of the axes at time $t + \delta t$ will be known from the displacements in time δt, due to these rotations, of the points $A\,(\omega_1, 0, 0)$, $B\,(0, \omega_2, 0)$, $C\,(0, 0, \omega_3)$. The components of these displacements in the directions OA, OB, OC, respectively, are evidently

$$\begin{array}{cccl} 0, & \omega_1\omega_3\delta t, & -\omega_1\omega_2\delta t, & \text{for } A \\ -\omega_2\omega_3\delta t, & 0, & \omega_2\omega_1\delta t, & \text{for } B \\ \omega_3\omega_2\delta t, & -\omega_3\omega_1\delta t, & 0, & \text{for } C. \end{array}$$

The component rotations at time $t + \delta t$ are $\omega_1 + \frac{d\omega_1}{dt}\,\delta t$, &c., which may be represented by OA', OB', OC'. The changes of the rotations in time δt are, therefore, AA', BB', CC'. Resolving these changes into the components (AF, FP, PA'), (BG, GQ, QB'), (CH, HR, RC') in the directions of the axes at time t, we get (observing that FP, PA' are the displacements in time δt of the point $F\,(\omega_1 + \frac{d\omega_1}{dt}\,\delta t, 0, 0)$, &c., and neglecting infinitesimals of the first order) the following as the resultant changes in time δt:

$$AF + GQ + HR = \left(\frac{d\omega_1}{dt} - \omega_2\omega_3 + \omega_3\omega_2\right)\delta t = \frac{d\omega_1}{dt}\,\delta t \text{ along } OA$$

$$FP + BG + RC' = \left(\omega_1\omega_3 + \frac{d\omega_2}{dt} - \omega_3\omega_1\right)\delta t = \frac{d\omega_2}{dt}\,\delta t \text{ along } OB$$

$$PA' + QB' + CH = \left(-\omega_1\omega_2 + \omega_2\omega_1 + \frac{d\omega_3}{dt}\right)\delta t = \frac{d\omega_3}{dt}\,\delta t \text{ along } OC.$$

The changes per unit time are therefore $\frac{d\omega_1}{dt}$, $\frac{d\omega_2}{dt}$, $\frac{d\omega_3}{dt}$ in the directions OA, OB, OC, respectively.

2. To determine the component changes of the body's moment of momentum. At time t the components of the moment of momentum are $A\omega_1$, $B\omega_2$, $C\omega_3$ in the directions of the principal axes, (where A, B, C denote the principal moments of inertia. At time $t + \delta t$ the components are $A\left(\omega_1 + \frac{d\omega_1}{dt}\,\delta t\right)$, &c., in the directions OA', OB', OC'. Employing the figure in a new sense, the former components may be represented by OA, OB, OC and the latter by OA', OB', OC'. The changes of the moment of momentum in time δt are therefore AA', BB', CC'. Resolving these changes into their components parallel to the axes at time t we get, as in the former case, (observ-

ing that FP, PA' are now the displacements in time δt of the point F, $\left(A\omega_1 + A\frac{d\omega_1}{dt}.\delta t,\ 0,\ 0\right)$, &c.), the following as the resultant changes of the moment of momentum in time δt:

$$\left(A\frac{d\omega_1}{dt} - B\omega_2\omega_3 + C\omega_3\omega_2\right)\delta t \text{ along } OA$$

$$\left(A\omega_1\omega_3 + B\frac{d\omega_2}{dt} - C\omega_3\omega_1\right)\delta t \text{ along } OB$$

$$\left(-A\omega_1\omega_2 + B\omega_2\omega_1 + C\frac{d\omega_3}{dt}\right)\delta t \text{ along } OC.$$

The changes per unit time are therefore $A\frac{d\omega_1}{dt} - (B - C)\,\omega_2\omega_3$, &c., in the directions OA, OB, OC, respectively.

21st November, 1878.

IV.

Condition of a Straight Line Touching a Surface.

By James Loudon, *University College, Toronto, Canada.*

The following method of getting the condition of a straight line touching a surface I have used for some years, and expected to see it in Salmon's 3d edition. It was so short and unimportant that I never thought of publishing it. (See Salmon's *Geometry of Three Dimensions, Art.* 80.)

Let $u \equiv ax^2 + by^2 + cz^2 + dw^2 + 2fyz + 2gzx + 2hxy + 2lxw + 2myw + 2nzw = 0$, then $P \equiv \alpha x + \beta y + \gamma z + \delta w = 0$, $P' \equiv \alpha' x + \beta' y + \gamma' z + \delta' w = 0$ is a line touching $u = 0$. Therefore $P - \lambda P' = 0$ is, for some value of λ, the tangent plane at the point of contact (x', y', z', w'); *i. e.*

$(\alpha - \lambda\alpha')\,x + (\beta - \lambda\beta')\,y + (\gamma - \lambda\gamma')\,z + (\delta - \lambda\delta')\,w = 0$ and
$(ax' + hy' + gz' + lw')\,x + (hx' + by' + fz' + mw')\,y + (gx' + fy' + cz' + nw')\,z$
$+ (lx' + my' + nz' + dw')\,w = 0$ are identical. Therefore

$$\begin{aligned} ax' + hy' + gz' + lw' &= k\,(\alpha - \lambda\alpha') & hx' + by' + fz' + mw' &= k\,(\beta - \lambda\beta') \\ gx' + fy' + cz' + nw' &= k\,(\gamma - \lambda\gamma') & lx' + my' + nz' + dw' &= k\,(\delta - \lambda\delta') \\ \alpha x' + \beta y' + \gamma z' + \delta w' &= 0 & \alpha' x' + \beta' y' + \gamma' z' + \delta' w' &= 0\,. \end{aligned}$$

Eliminating x', y', z', w', $-k$, $k\lambda$, we get the required condition

$$\begin{vmatrix} a, & h, & g, & l, & \alpha, & \alpha' \\ h, & b, & f, & m, & \beta, & \beta' \\ g, & f, & c, & n, & \gamma, & \gamma' \\ l, & m, & n, & d, & \delta, & \delta' \\ \alpha, & \beta, & \gamma, & \delta, & 0, & 0 \\ \alpha', & \beta', & \gamma', & \delta', & 0, & 0 \end{vmatrix} = 0\,.$$

28th November, 1878.